ticularly important for applications to experiment.

The angle dependence of transition fields can be obtained for comparison with experiments on single crystals, and if only the c axis of the sample is defined, comparisons with experiment may be made by numerical averaging of the solutions over all in-plane angles. The material VF_2,[13,14] a planar spiral with $q_0 = 96°$, should be a good subject for this type of study. High-field magnetization ($H \perp c$ axis) at $T \ll T_N = 7$ °K on this compound would permit an instructive application of the theory here described; behavior similar to that depicted in Fig. 4 should be found, if as reported[13] the in-plane anisotropy is uniaxial and small.

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Metallic Alloys and Exchange-Enhanced Paramagnetism. Application to Rare-Earth–Cobalt Alloys

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A phenomenological theory is presented of the magnetic properties of two-component alloys, one of whose components has a permanent magnetic moment, and the other an exchange-enhanced paramagnetic susceptibility. The paramagnetic susceptibility, magnetic-ordering temperature, and magnetization at low temperature are discussed in terms of this theory, and the results obtained are applied to cubic alloys ACo_2 between rare earth (A) and cobalt.

INTRODUCTION

The behavior of intermetallic alloys between rare earths and transition metals is often complex.[1] Cobalt does not possess a magnetic moment in YCo_2 or $LuCo_2$. In the alloys with magnetic rare earths, the magnetic moment of cobalt varies with the spin of the rare earth. In $GdCo_2$ or $TbCo_2$, the magnetic moments of the rare-earth and cobalt atoms are antiparallel, whereas in $NdCo_2$ or $PrCo_2$ they are ferromagnetically aligned.[2]

We present a phenomenological theory for these alloys, and using this theory we study their paramagnetic susceptibility, magnetic-ordering temperature, and magnetization at low temperature.

I. MODEL AND ITS MAIN CONSEQUENCES

We consider an alloy formed with two types of atoms, A and B, located in two different crystallographic sites. We assume that A possesses a well-localized magnetic moment and that B gives rise, in the crystal, to electronic energy bands leading to an exchange-enhanced paramagnetic susceptibility. In the high-temperature range, the magnetizations of A and B atoms, in an applied magnetic field H, are

$$M_A=(C_A/T)(H+n_{AA}M_A+n_{AB}M_B) , \quad (1)$$

$$M_B=\chi_{B,0}(H+n_{BB}M_B+n_{AB}M_A) , \quad (2)$$

where $\chi_{B,0}$ is the paramagnetic susceptibility; C_A is the Curie constant of the A atoms; and n_{AA}, n_{BB}, and n_{AB} are molecular-field coefficients which represent exchange interactions inside A and B sublattices and between these two sublattices. When the magnetization of the A atoms is equal to zero, then the total susceptibility is the exchange-enhanced susceptibility

$$\chi_y=M_B/H=\chi_{B,0}/(1-n_{BB}\chi_{B,0}) . \quad (3)$$

From Eqs. (1)–(3) one obtains for the susceptibility of the sample,

$$\chi=[C_A+\chi_y(T-EC_A)]/(T-\Theta_B) , \quad (4)$$

with

$$E=n_{AA}-2n_{AB} . \quad (5)$$

The magnetic-ordering temperature Θ_B is given by

$$\Theta_B=(n_{AA}+n_{AB}^2\chi_y)C_A . \quad (6)$$

In the neighborhood of Θ_B, in the magnetically ordered domains, the spontaneous magnetization of the B atoms is proportional to that of the A atoms, according to the relation

$$M_B=n_{AB}\chi_y M_A . \quad (7)$$

At low temperature, the magnetization M_B cannot be considered as proportional to the magnetization M_A, because of the variation of χ_y with the magnetization M_B itself.

II. PARAMAGNETIC PROPERTIES OF ACo_2 ALLOYS

Equation (4) for the paramagnetic susceptibility contains two parameters, E and χ_y; Θ_B is the experimentally determined magnetic-ordering temperature. As a first approximation, these parameters are assumed to be temperature independent; $1/\chi$ as a function of temperature (3) for various ACo_2 alloys is shown in Fig. 1. The variation calculated from Eq. (4) with the E and χ_y parameters, as given in Table I, is shown in the same figure.

We see that two parameters are sufficient to give a good fit to the experimental curves. The χ_y values obtained agree with the room-temperature values of the paramagnetic susceptibility of nonmagnetic alloys YCo_2 (39×10^{-4} emu/mole),[1] $ScCo_2$ (20×10^{-4} emu/mole),[3] and $LuCo_2$ (30×10^{-4} emu/mole).[4]

In YNi_2, $ScNi_2$, or $LuNi_2$, the $3d$ electronic band is full, which give rise to a small susceptibility of the order of 3×10^{-4} emu/mole [3,5]; in YFe_2 or $LuFe_2$ the density of states at the Fermi level is large enough for the Stoner condition for ferromagnetism to be satisfied. In these compounds, iron possesses a magnetic moment of $1.45\mu_B$.[6]

From Eqs. (5) and (6), we can deduce the values of n_{AA} and n_{AB} (Table I). It is more meaningful, however, to consider exchange-interaction coefficients J_{AA} and J_{AB}, since they are not spin depen-

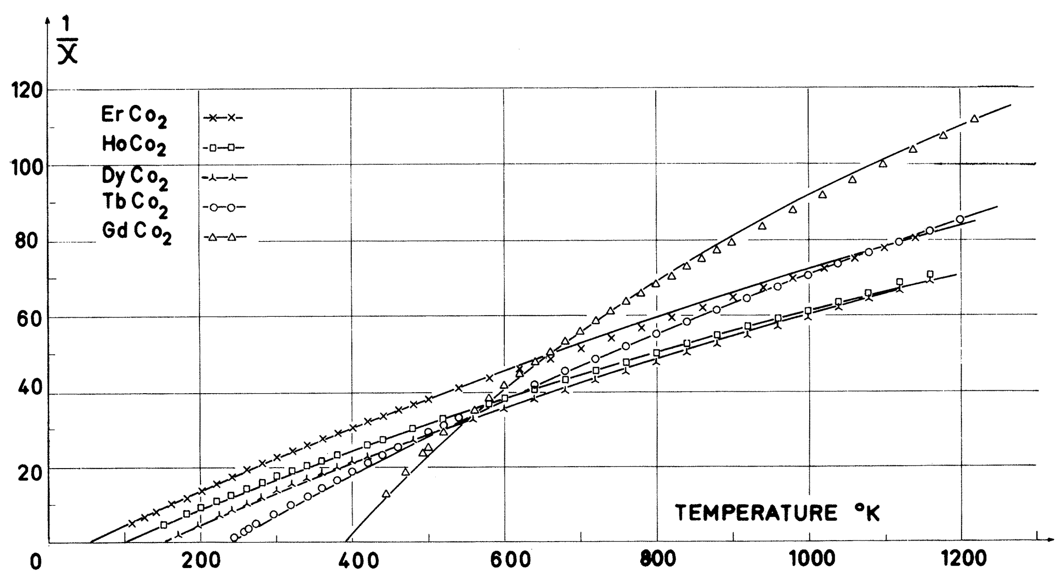

FIG. 1. Thermal variation of the inverse paramagnetic susceptibility of various ACo_2 alloys. Full curves are theoretical (see text). Data points are taken from Bloch *et al.* (Ref. 4).

TABLE I. Magnetic-ordering temperature, enhanced paramagnetic susceptibility, and main exchange-interaction coefficients of ACo_2 alloys.

	$\Theta_B(K)$	χ_y (10^{-4} emu/mole)	E	J_{AA}	$-J_{AB}$
$GdCo_2$	395	36	170	60	−140
$TbCo_2$	240	28	119	58	−159
$DyCo_2$	150	35	78.5	66	−140
$HoCo_2$	100	39	60	52	−140
$ErCo_2$	60	33	40	66	−120

dent. Their values are given by

$$J_{AB}=g_J n_{AB}/(g_J-1) \tag{8}$$

and

$$J_{AA}=(g_J)^2 n_{AA}/2(g_J-1)^2, \tag{9}$$

where g_J is the Landé factor associated with the J quantum number of the A rare earth.

The values J_{AA} and J_{AB} thus obtained (Table I) are homogeneous.

III. MAGNETIC-ORDERING TEMPERATURES AND MAGNETIZATION

The magnetic-ordering temperature Θ_B [Eq. (6)] can be considered as the sum of two contributions. One, $n_{AA}C_A$, comes from exchange interactions between the A atoms; the other, $n_{AB}^2 C_A \chi_y$, is a contribution from the B atoms. The two contributions are approximately equal in ACo_2 alloys. A comparison with the Curie temperature of ANi_2 alloys[5] indicates that rare-earth–rare-earth interactions are two times more important in ACo_2 than in ANi_2, although the interatomic distances are almost identical. This indicates that the electrons of the incomplete $3d$ band participate in rare-earth–rare-earth exchange interactions.

When the magnetic moment of the transition metal is weak, one can consider its value as given by Eq. (6), which can be written

$$M_B=\tfrac{1}{2}J_{AB}\chi_y S_A .$$

In agreement with the experimental results, the magnetic moment of the transition metal is a function of the alloyed rare earth. It is equal to zero for YCo_2, $LuCo_2$, or $ScCo_2$, and has its maximum value for $GdCo_2$.

If we take the values of $J_{AB}=-150$ and $\chi_y=35\times10^{-4}$ (Table I), we obtain for cobalt in $GdCo_2$ a calculated magnetization at absolute zero of $0.91\mu_B$ compared with the experimental value $1.05\mu_B$; and for Co in $PrCo_2$, we obtain $0.26\mu_B$ compared with the experimental value [7] $(0.50\pm0.25)\mu_B$.

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